Regular truss structure model equivalent to continuum structure

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Abstract: It is difficult to solve the structural problems related to agricultural engineering, due to the wide ranges of the means of related variables and complex structural shapes. For these reasons, discrete models are required that are able to replace or simplify solid structure components used in traditional analysis methods. Therefore, the objective of this study was to develop a regular truss structure model that behaves the same way as a solid structure. It was assumed that if a unit element consists of truss elements with each hinge at the end of the element and the size of the element is infinitesimal, the stress distribution and displacement field will be constant throughout the domain of the unit element. Additionally, the behavior of the truss element was assumed to be in a linear state in a two-dimensional plane. The law of energy conservation, based on the theory of elasticity, was applied to determine the equilibrium conditions between discretized and solid elements. The restrictive condition that we obtained revealed that applications are limited to only ideal elastic materials with a Poisson's ratio of 1 to 3. The volumetric ratio of the equivalent truss to the continuum structures was 3:1, regardless of the size or number of the mesh. To calculate the internal stress and strain of the unit element, the geometric relationships of each truss member, which has its own role against different stress directions, were used. The calculated von Misses stresses were used to verify this model. Stress concentrations, as explained based on Saint Venant's principle, were also observed in the equivalent truss structure model. The main stress paths, indicating the areas where reinforcement bars should be placed, were successfully shown without the requirement that each element be transformed in the direction of principal stress; this was done by eliminating elements with only compressive and near-zero stresses.

Keywords: structural analysis, regular mesh, equivalent truss structure model, discretized element, energy method
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1 Introduction

Traditional structure analysis methods lead to longer computational times because they use solid elements in order to enable numerous degrees of freedom. Continuum models also require complex processes to execute reliability analyses because there is no way to automatically calculate failure modes. In addition, continuum models have difficulty expressing crack propagation because crack propagation is governed by a discrete fracture mechanism, whereas solid elements are based on continuum theory. To overcome these shortcomings, a few equivalent structure models that have the same behavior as continuum structures have been developed using discretized elements.

In the last two decades, equivalent truss structure models have been used to execute topology designs of two-dimensional continuum structures. In these studies, the continuum that was used to show elastic behavior was discretized by regular lattice unit cells suggested by the

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so-called cellular automata (CA) concept. The lattice cell consisted of one node at the center of the cell surrounded by eight additional nodes, as suggested by Moore's neighborhood rule[7,12]. To maintain the balance of the system, each cell transfers its redundant value to its neighbors, as dictated by the local rules, which define the relationship between the center cell and its neighboring cells. There was no need to set up a governing equation to satisfy the whole domain. Therefore, cellular automata could be used as an alternative for simulating unknown physical phenomena.

Earlier applications of cellular automata mainly concentrated on shape optimization[2,4,5,13,15]. The reference values or ratios were calculated and updated at each designated cell by applying local rules. However, since the local rules were determined by numerical experiences, the relationship between the mathematical rule and topology optimization can change depending on designer opinion. Other approaches have used the traditional Moore's neighborhood, which is split into four regularly spaced quadrants without connectivity in the diagonal directions[1]. These design and analysis rules were derived based on the continuous optimality criteria, which were interpreted as the local Kuhn-Tucker conditions and the principle of minimum total potential energy, respectively. The topology was regularized based on the simple isotropic microstructure with penalization (SIMP) method for each time step[6]. In the literature, an evolutionary structural optimization (ESO) technique has been applied to automatically build up the strut-and-tie model using its own terminal criteria, and a finite element method (FEM) based on the plane stress element has been used for design and analysis. The literature has also introduced a special condition (the so-called CA constraint) to minimize the variation of the equivalent stress of neighboring cells with respect to variation in the thickness of the updated cell.

In order to improve the idea of discretization, which is one of the merits of cellular automation, small square cells were replaced by equivalent truss structure units, and the discretized structure was analyzed by repeating the static equilibrium condition at each node throughout the whole domain. The stress ratio (SR) method, using von Misses stress, was applied to the design rule. For these reasons, this method can adopt a parallel computing system, which significantly decreases computational time. This is accomplished without having to create a global stiffness matrix, which was a requirement of previous methods[10]. Another study applied the cellular automata paradigm and the equivalent truss model to the geometric nonlinear topology design of continuum structures[14]. This heuristic approach, changing the density interpolation scheme in order to consolidate the void and solid regions, has been regularly employed to prevent a checkerboard pattern from occurring during simulation. In other research, geometric and geometric/material nonlinear topology designs of truss structures have been explored[3,8]. The SIMP technique was expanded to the simultaneous fiber path and topology design of anisotropic lamina in a cellular automata framework[9]. Displacements were sequentially updated to satisfy the local equilibria of CA cells. Fiber angles and density measurements were also updated based on the optimality criteria for a minimum compliance design. The hybrid cellular automaton (HCA) methodology, inspired by the biological process of bone remodeling, was developed for both topology and shape optimization of continuum structures[11]. In bone remodeling, cells with low elastic modulus values are removed and nearby cells create a new cell in the empty surrounding space. This approach creates structures that are similar to the ones observed in bird bones.

However, most previous studies have concentrated only on topology optimization paradigms (based on cellular automata) as opposed to attempting to improve the truss structure model that is equivalent to the continuum. Therefore, the objective of this investigation was to develop an advanced regular truss structure unit where each member had only hinges at both ends of the element; the analysis results were validated by comparison with the continuum solid model and used to arrange reinforcement bars in a practical manner.

2 Basic assumptions of discretized models

2.1 Relationship between stress and strain

Normal and shear stresses should be taken into consideration simultaneously for a discretized model under plane stress. The unit under normal stresses shows linear-elastic behavior in the direction of the normal stresses as well as deforms in a direction perpendicular to the normal stresses. Most materials that have isotropic elastic properties are subject to the transverse deformation in accordance with Poisson's ratio when normal stresses are applied. These units can also experience coplanar stresses, referred to as shear stresses, parallel to the cross-section on which the normal stress is applied. This deformation is related to a change in the angle at the vertex. Therefore, the normal and shear stresses should be included in any discretized unit under isotropic elastic conditions, as shown in Figure 1.

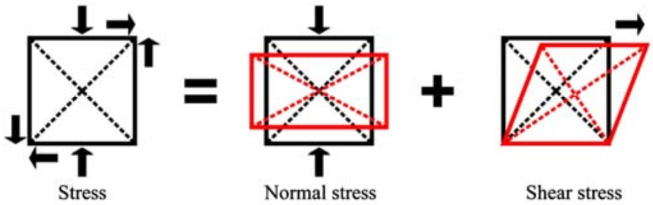


Figure 1 Diagonal deformation by stresses

2.2 Basic shape of discretized model

Consideration may be given to the use of truss elements that are able to discretize a continuum structure. The discretized basic unit consists of truss elements that should be structurally stable for building a certain structure. A triangle is suitable as the basic shape of a discretized model because it is stable in itself and can also be easily expanded into various shapes. Additionally, discretized units can be made up of combinations of any triangular shape, serving as alternatives if the stable conditions are structurally satisfied, as shown in Figure 2.

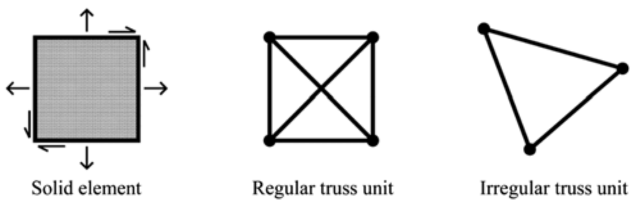


Figure 2 Feasible shapes of discretized units equivalent to a solid element

2.3 Discretized triangular unit

A great deal of research has been done to create efficient mesh shapes that contribute to mesh quality; the generation of an appropriate mesh type guarantees accurate analysis results in any related problems. Among all shapes, a triangular element is generally utilized as the basic unit and can maximize the effectiveness of structural modeling with fewer elements. However, the unit element of a solid structure assumes the rectangular shape as its basic shape so that normal and shear stresses can be expressed symmetrically and simultaneously. In order to discretize the normal and shear stresses that occur in the element into the equivalent member forces of any triangular truss unit, an exact understanding between the rectangular solid element and the triangular truss unit is required (Figure 3).

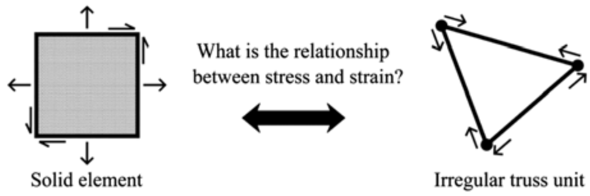


Figure 3 Unknown relationship between the solid element and the irregular truss unit

Therefore, the following transformation procedures are suggested for the construction of triangular truss meshes in the domain. First, the normal and shear stresses are computed via structural analyses based on continuum mechanics. Second, the principal stresses are calculated such that the normal and shear stresses are expressed only as tensile and compressive stresses. Truss members are laid in sequence in the directions of the principal stresses, as shown in Figure 4. Finally, analysis of the truss structure is carried out (as opposed to analysis of the continuum structure). Unfortunately, this method requires continuum analysis in order to complete equivalent truss structure analysis because it necessitates the use of an automated shape-optimization modeling technique to arrange the truss members in the same direction, which is equivalent to the principal stresses. Additionally, continuum analysis must be continually repeated whenever the shape of the structure changes; this is due to the occurrence of partial destruction, large deformations, and other factors. Therefore, there is a limit to its general application because it demands complicated and unformulated processes when the continuum structure is transformed into the equivalent truss structure, which is made up of only tensile and compressive forces.

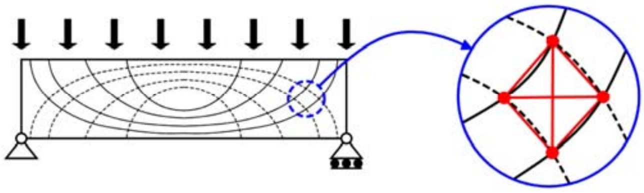


Figure 4 Irregular truss unit that is ideally stationed depending on the principal stresses

2.4 Discretized rectangular unit

The unit used to replace the solid element is assumed to be a rectangular shape that can express the normal and shear stresses symmetrically and simultaneously. The rectangular shape of a truss unit, which behaves as an infinitesimal element of a continuum structure, was studied[10]. The interactive relationship was derived based on the assumption that the amount of energy changed by the normal stresses in an infinitesimal element is identical to the amount of energy changed by the vertical and horizontal members of the truss unit. Additionally, the amount of energy changed by the shear stress of the element is identical to the amount of energy changed by the diagonal members of the truss unit. However, this study showed inconsistencies between the original shape of the truss unit and the shapes formed by overlapping areas of truss units, as shown in Figure 5.

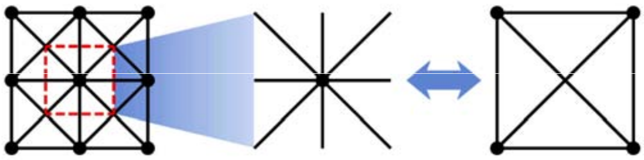


Figure 5 Dissimilarity of neighboring shapes

3 Mathematical model

3.1 Definition of regular truss unit

The enhanced equivalent regular truss unit was redefined, as shown in Figure 6, such that the shape formed by the neighboring areas of the truss unit maintained an identical shape with the truss unit in all of the relevant areas. It is assumed that the normal stresses and shear stress are related to the horizontal and vertical truss members, and the diagonal truss members, respectively. ε_{d1} and ε_{d2} indicate the diagonal strains of each infinitesimal element when both normal and shear stresses are applied to the element. Details of the sectional areas and strains are shown in Figure 7.

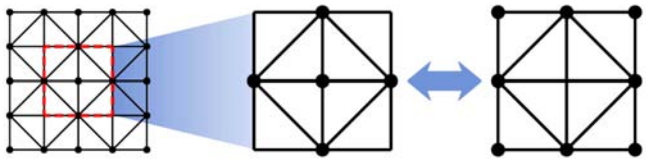


Figure 6 Enhanced regular truss unit and its similarity with neighboring shapes

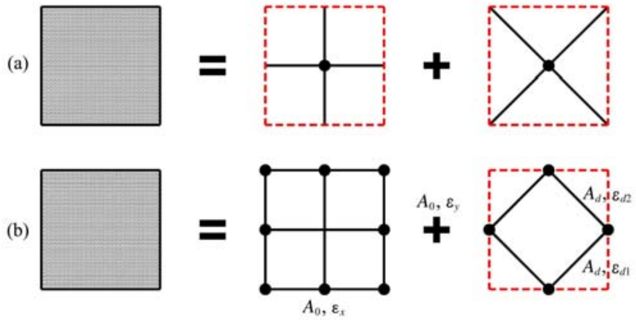


Figure 7 Comparison of (a) existing and (b) enhanced truss units under normal and shear strains

3.2 Restricted conditions

The principle of the conservation of energy, which states that the internal energy of the infinitesimal element is equal to the sum of the energies of each member of the regular truss unit, was adopted to determine the compatibility conditions. The formulated processes of this methodology are described below. The internal energy of the infinitesimal element under two-dimensional plane stress can be expressed in terms of the strains as:

$$U = \left[\frac{E}{1-\nu^2}(\varepsilon_x^2+\varepsilon_y^2+2\nu\varepsilon_x\varepsilon_y)+G\gamma_{xy}\right]\frac{\ell^2 t}{2} \tag{1}$$

where, U is the internal energy, J; E is Young's modulus, Pa; ν is Poisson's ratio; G is the shear modulus, Pa; γ_{xy} is the shear strain; ℓ is the length of one side of the solid element, m; t is the thickness of the solid element, m.

The internal energies of the horizontal, vertical, and diagonal members of the equivalent truss unit can be presented in terms of the strain as:

$$U_{TSEH} = 3\cdot\frac{1}{2}\sigma_x\varepsilon_x A_o\ell = \frac{3}{2}E\varepsilon_x^2 A_o\ell \tag{2}$$

$$U_{TSEV} = 3\cdot\frac{1}{2}\sigma_y\varepsilon_y A_o\ell = \frac{3}{2}E\varepsilon_y^2 A_o\ell \tag{3}$$

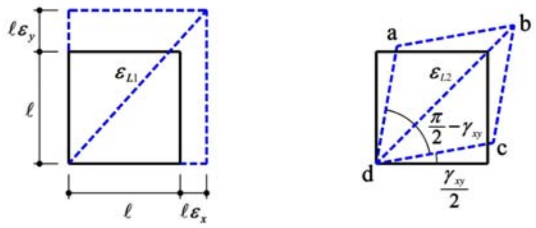
$$\begin{aligned} U_{TSED} &= 2\cdot\frac{1}{2}E\varepsilon_{d1}^2 A_d\frac{\sqrt{2}\ell}{2} + 2\cdot\frac{1}{2}E\varepsilon_{d2}^2 A_d\frac{\sqrt{2}\ell}{2} \\ &= \sqrt{2}A_d(\varepsilon_{d1}^2+\varepsilon_{d2}^2)\frac{E\ell}{2} \end{aligned} \tag{4}$$

where, A_o is the cross-sectional area of the horizontal and vertical members, m^2; A_d is the diagonal area of the diagonal members, m^2; σ_x and σ_y are the normal stresses in the x and y directions, Pa; U_{TSEH}, U_{TSEV}, and U_{TSED} are the internal energies of the horizontal, vertical, and diagonal members of the truss structure unit, respectively, J.

Here, each strain (ε_{L1} and ε_{L2}) that occurred in a diagonal direction (caused by the normal and shear stresses) is schematized, as shown in Figure 8. Each strain (ε_{d1} and ε_{d2}) in a diagonal direction, as shown in Figure 7, can be presented as:

$$\varepsilon_{d1} = \varepsilon_{L1} + \varepsilon_{L2} \tag{5}$$

$$\varepsilon_{d2} = \varepsilon_{L1} - \varepsilon_{L2} \tag{6}$$



a. Normal strain b. Shear strain

Figure 8 Deformation of an enhanced regular truss unit

In Equations (5) and (6), the strains ε_{L1} and ε_{L2} can be derived from the geometric conditions shown in Figure 8. First, ε_{L1} is defined as:

$$\varepsilon_{L1} = \frac{\sqrt{(\ell+\ell\varepsilon_x)^2 + (\ell+\ell\varepsilon_x)^2} - \sqrt{2}\ell}{\sqrt{2}\ell} = \sqrt{1+(\varepsilon_x+\varepsilon_y)} - 1 \tag{7}$$

Because both ε_x and ε_y are extremely small values, Equation (8) can be written as:

$$1+(\varepsilon_x+\varepsilon_y) = \left(1+\frac{\varepsilon_x+\varepsilon_y}{2}\right)^2 \tag{8}$$

Therefore, Equation (7) can be rearranged as:

$$\varepsilon_{L1} = \frac{\varepsilon_x+\varepsilon_y}{2} \tag{9}$$

From the geometric conditions in Figure 8, ε_{L2} can be defined as:

$$\varepsilon_{L2} = \frac{L_{bd} - \sqrt{2}\ell}{\sqrt{2}\ell} \tag{10}$$

where, L_{bd} is the diagonal length after deformation, m.

Using the trigonometric equation, L_{bd} can be rearranged as:

$$L_{bd}^2 = \ell^2 + \ell^2 - 2\ell^2\cos\left(\frac{\pi}{2}+\gamma_{xy}\right) \tag{11}$$

Equation (12) can be derived from Equations (10) and (11) as:

$$\varepsilon_{L2} = \frac{\gamma_{xy}}{2} \tag{12}$$

Finally, Equations (13) and (14) can be obtained by substituting Equations (9) and (12) into Equations (5) and (6), respectively.

$$\varepsilon_{d1} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\gamma_{xy}}{2} \tag{13}$$

$$\varepsilon_{d2} = \frac{\varepsilon_x+\varepsilon_y}{2} - \frac{\gamma_{xy}}{2} \tag{14}$$

Using the principle of the conservation of energy, which states that internal energy subjected to the infinitesimal element is always equal to the sum of the internal energies of all truss members (i.e., the horizontal, vertical, and diagonal members, as described above) of the equivalent truss structure unit, the following restricted conditions were induced:

$$A_o = \frac{1}{4}\ell t \quad A_d = \frac{3\sqrt{2}}{8}\ell t \quad \nu = \frac{1}{3} \tag{15}$$

3.3 Modeling

The method of modeling for the case, where members of the equivalent truss structure overlap, is shown in Figure 9. When the truss elements are joined together, the cross-sectional areas of the elements are doubled. However, the cross-sectional areas of the outermost elements are not doubled since they do not overlap with other elements. Because the equivalent truss structure behaves in the same way as the continuum structure, the equivalent nodal loads (which are the point loads transformed from the continuous loads acting on the boundaries) should be applied to the equivalent truss structure. Therefore, the external loads are converted in accordance with the three-point Gaussian integration rule; the equivalent truss structure unit has three nodes on one side, as shown in Figure 10, which depicts the uniform load case.

3.4 Stress and strain

In the equivalent truss unit, the force in the direction of the x-axis is taken by the horizontal and diagonal members, the force in the direction of the y-axis is taken by the vertical and diagonal members, and the shear force is taken only by the diagonal members, as shown in

Figure 11.

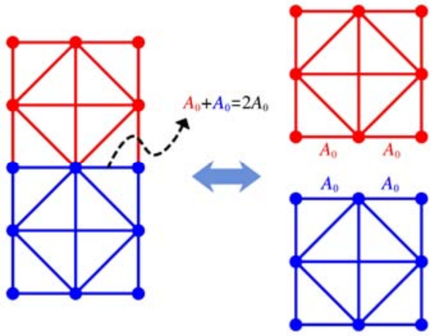
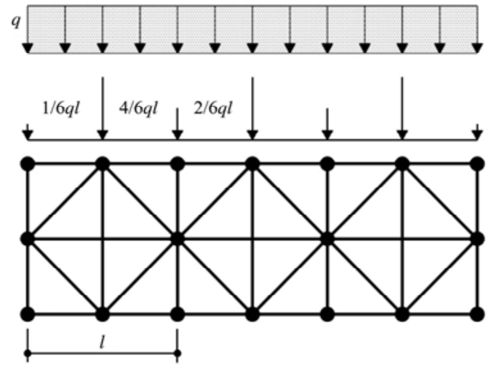
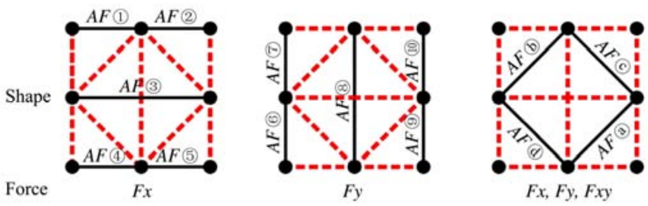


Figure 9 Modeling of enhanced regular truss unit in an overlapped area



Note: q is the uniform distributed load, N/m; l is the length of one side of the unit element, m.

Figure 10 Equivalent nodal loads by three-point Gaussian integration rule



Note: $A.F$①-⑤, $A.F$⑥-⑩ and $A.F$ⓐ-ⓓ are related with the normal stresses in the x and y directions, respectively, and the normal and shear stresses.

Figure 11 Truss elements related with external forces

When shear forces occur, members ⓐ and ⓑ affect the shear force in the positive direction, whereas members ⓒ and ⓓ affect the shear force in the negative direction. Therefore, members ⓐ and ⓑ are marked with positive signs, whereas members ⓒ and ⓓ are marked with negative signs. The summations of the forces applied to the equivalent truss unit are derived as:

$$F_x=\left(\frac{A.F\text{①}}{2}+\frac{A.F\text{②}}{2}\right)\frac{1}{2}+A.F\text{③}+\left(\frac{A.F\text{④}}{2}+\frac{A.F\text{⑤}}{2}\right)\frac{1}{2}+ \left(A.F\text{ⓐ}+A.F\text{ⓑ}\right)\frac{1}{2}\frac{1}{\sqrt{2}}+\left(A.F\text{ⓒ}+A.F\text{ⓓ}\right)\frac{1}{2}\frac{1}{\sqrt{2}} \quad (16)$$

$$F_y=\left(\frac{A.F\text{⑥}}{2}+\frac{A.F\text{⑦}}{2}\right)\frac{1}{2}+A.F\text{⑧}+\left(\frac{A.F\text{⑨}}{2}+\frac{A.F\text{⑩}}{2}\right)\frac{1}{2}+ \left(A.F\text{ⓐ}+A.F\text{ⓑ}\right)\frac{1}{2}\frac{1}{\sqrt{2}}+\left(A.F\text{ⓒ}+A.F\text{ⓓ}\right)\frac{1}{2}\frac{1}{\sqrt{2}} \quad (17)$$

$$F_{xy}=\left(A.F\text{ⓐ}+A.F\text{ⓑ}\right)\frac{1}{2}\frac{1}{\sqrt{2}}-\left(A.F\text{ⓒ}+A.F\text{ⓓ}\right)\frac{1}{2}\frac{1}{\sqrt{2}} \quad (18)$$

To calculate the forces in the outermost members, the one half constant values in Equations (16)-(18) should be eliminated because these members do not have neighboring members to share the forces. The corresponding stresses can be obtained by using the forces and sectional areas, and the strains can be computed from the relationship between the stress and strain. Because the model developed in this study is an approximate analysis model, it was found that more accurate shear strains can be provided from the axial forces as opposed to from the stress-strain relationship that is shown in Equation (19). However, any of these methods can be used because they all provide solutions that are within the reasonable range of accuracy (with the exception of the unconcerned area).

$$\varepsilon_{d1}=\left(\frac{A.F\text{ⓐ}+A.F\text{ⓑ}}{2A_dE}\right)\quad \varepsilon_{d2}=\left(\frac{A.F\text{ⓒ}+A.F\text{ⓓ}}{2A_dE}\right)$$
$$\gamma_{xy}=\varepsilon_{d1}-\varepsilon_{d2} \quad (19)$$

4 Results and discussion

4.1 Deflection

Vertical deflections, which occurred at the bottom center of simply supported beams under a concentrated load, were examined. The beams included the following details: a width of 0.2 m, depth of 0.5 m, elastic modulus of 199.81 GPa, concentrated load of 980 kN, and spans varying from 2.0 m to 4.5 m. To check the degrees of accuracy, the results of the vertical deflections were compared with the solutions from the finite element method (FEM) that was based on three-dimensional solid elements. The number of square grids in the height direction was fixed at 20 for both methods and the number of square grids in the span direction was varied in accordance with the ratio of the span length to the span height. The results in Table 1 show that the relative error between the equivalent truss structure model and the three-dimensional FEM was 3.28%, even when the ratio of the length to the height was four. As the ratio was increased, it was observed that the relative error was reduced.

Table 1 Deflection of a simply supported beam under a concentrated load Unit: mm

Length/height	RTSM⁺	FEM⁺⁺	Error⁺⁺⁺/%
4	5.44057	5.26228	3.28
5	9.39066	9.14877	2.58
6	15.20773	14.87448	2.19
7	23.26511	22.84726	1.80
8	33.93622	33.31500	1.83
9	47.59453	46.76543	1.74

Note: ⁺: Regular truss structure model (RTSM) used in this study; ⁺⁺: FEM using 3D solid elements; ⁺⁺⁺: Relative error.

A previous truss structure model[10] was compared with the truss structure model developed in this study; comparisons were made in terms of the vertical deflections at the bottom center of the beam (with a span of 2.5 m) as the number of members was increased. The state of plane stress can be assumed for the continuum analysis, i.e., the stress perpendicular to the mid-plane is zero. A simply supported beam loaded in a two-dimensional plane can be used as a representative example because the stress related to the beam width is negligible. However, because deformation in the direction of the beam's width is not constrained, problems using three-dimensional solid structure analysis without constraints in the direction of the beam's width can also be solved. Therefore, the results of the truss structure model were also compared with those solved using the FEM based on three-dimensional solid elements. The truss structure model proposed in this study assumed that its basic unit behaves equivalently to infinitesimal solid elements in the state of plane stress. The deflection curve of the equivalent truss structure model was closer to the solid structure model without any constraint in the direction of the z-axis than it was with the structure model that had constraints, as shown in Figure 12.

As the number of elements increased, the deflection curve obtained by the previous truss structure model[10] became less similar to the exact solution, whereas the results of the enhanced truss structure model became more similar to the exact solution. Although the analysis was done with a small number of truss elements, the enhanced equivalent truss structure model provided a fast convergence rate.

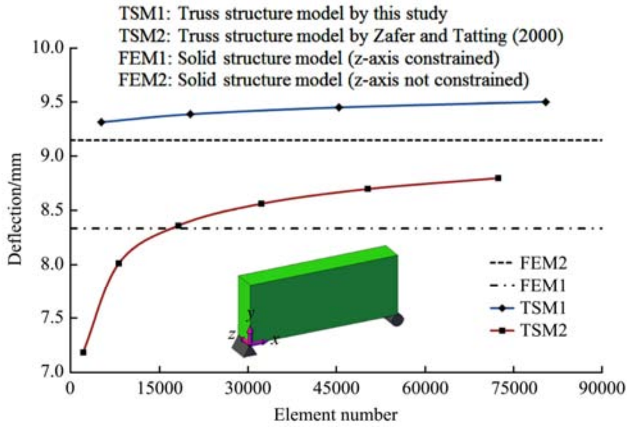


Figure 12 Deflections of the regular truss units as a function of the number of truss elements

4.2 Stress and strain

The stresses and strains of a simply supported beam under a uniform load were reviewed at the center of the beam. The details of the beam are: a width of 0.02 m, depth of 0.5 m, elasticity modulus of 199.81 GPa, uniform load of 9.8 MN/m, and span of 2.5 m. The meshing contained 20 grids for the height and 100 grids for the length. To determine the degree of accuracy, the results of the stresses and strains were compared with those determined by the FEM based on three-dimensional solid elements without constraints in the direction of the beam's width. It was found that the degrees of accuracy of the solutions were less than 5% of the relative error in most locations, as shown in Tables 2 and 3.

Table 2 Sectional stresses of a simply supported beam under a distributed load

Y	σ_x		σ_y		τ_{xy}	
Coordinate from bottom/mm	Stress/GPa	Error/%	Stress/GPa	Error/%	Stress/GPa	Error/%
12.5	8.42736	−4.21	−0.00159	76.56	0.00172	−3.73
37.5	7.91887	1.08	−0.00818	4.17	0.00498	−2.18
62.5	6.98414	1.47	−0.02116	0.53	0.00797	−0.81
87.5	6.03219	1.50	−0.03976	−0.01	0.01058	−0.20
112.5	5.08812	1.51	−0.06321	−0.07	0.01282	0.10
137.5	4.15214	1.52	−0.09076	−0.02	0.01469	0.27
162.5	3.22287	1.53	−0.12167	0.04	0.01618	0.37
187.5	2.29881	1.55	−0.15518	0.10	0.01729	0.40
212.5	1.37846	1.60	−0.19054	0.14	0.01803	0.40
237.5	0.46033	1.84	−0.22701	0.16	0.01839	0.37
262.5	−0.45707	1.12	−0.26382	0.18	0.01838	0.32
287.5	−1.37525	1.36	−0.30025	0.18	0.01800	0.25
312.5	−2.29569	1.41	−0.33554	0.17	0.01725	0.15
337.5	−3.21988	1.43	−0.36895	0.16	0.01612	0.02
362.5	−4.14930	1.45	−0.39974	0.13	0.01463	−0.16
387.5	−5.08543	1.45	−0.42716	0.10	0.01276	−0.40
412.5	−6.02964	1.46	−0.45049	0.06	0.01052	−0.79
437.5	−6.98179	1.43	−0.46897	0.01	0.00792	−1.44
462.5	−7.91814	1.07	−0.48187	−0.06	0.00495	−2.83
487.5	−8.45111	−3.94	−0.48841	−0.14	0.00171	−4.38

Table 3 Sectional strains of a simply supported beam under a distributed load

Y	ε_x		ε_y		γ_{xy}	
Coordinate from bottom/mm	Strain	Error/%	Strain	Error/%	Strain	Error/%
12.5	0.04217	−4.21	−0.01266	−4.18	0.00002	−1.26
37.5	0.03964	1.08	−0.01193	1.09	0.00006	0.33
62.5	0.03498	1.47	−0.01059	1.46	0.00010	1.73
87.5	0.03024	1.50	−0.00925	1.47	0.00014	2.35
112.5	0.02555	1.50	−0.00795	1.44	0.00017	2.67
137.5	0.02091	1.51	−0.00668	1.41	0.00019	2.85
162.5	0.01631	1.51	−0.00544	1.36	0.00021	2.94
187.5	0.01173	1.52	−0.00422	1.28	0.00023	2.98
212.5	0.00718	1.54	−0.00302	1.13	0.00024	2.98
237.5	0.00264	1.62	−0.00182	0.79	0.00024	2.95
262.5	−0.00189	1.32	−0.00063	−0.82	0.00024	2.90
287.5	−0.00643	1.45	0.00056	4.66	0.00024	2.82
312.5	−0.01098	1.47	0.00176	2.62	0.00023	2.72
337.5	−0.01556	1.48	0.00298	2.24	0.00021	2.58
362.5	−0.02016	1.49	0.00422	2.08	0.00019	2.40
387.5	−0.02480	1.49	0.00549	1.99	0.00017	2.15
412.5	−0.02950	1.49	0.00679	1.93	0.00014	1.76
437.5	−0.03423	1.46	0.00813	1.85	0.00010	1.09
462.5	−0.03890	1.09	0.00947	1.36	0.00006	−0.34
487.5	−0.04156	−4.00	0.01024	−4.80	0.00002	−1.93

When the computed stress and strain values were relatively low, the relative errors were high and the values were not sufficient to obtain a solution. However, the low values in the structural analysis are not a design concern; therefore, we believe that the equivalent truss structure model provides a sufficient number of solutions for practical purposes.

4.3 Volumetric ratio

The consistency of the volumetric ratio of the previous truss structure model[10] was compared with the equivalent truss structure model that is proposed in this study. The changes in the volumetric ratios, as a factor of the number of truss members, were observed for the simply supported beam under a concentrated load. Both models maintained a volumetric ratio of 3 to 1 in each unit. However, this situation changed when the units were combined into a structure. The volumetric ratio of the previous truss structure model approached 3 to 1 as the total number of truss members increased, whereas the volumetric ratio of the equivalent truss structure model was constant at 3 to 1, as shown in Figure 13. This indicates that the equivalent truss structure model is more consistent for structural analysis.

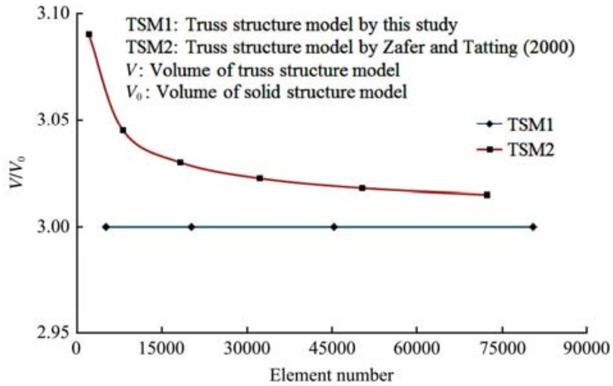


Figure 13 Volume ratios of the regular truss units as a function of the number of truss elements

4.4 von Misses stress

To check the von Misses stress, a deep beam (shown in Figure 14a) was used. This beam had a width of 0.01 m, depth of 0.25 m, span of 0.5 m, elasticity modulus of 200 GPa, and sine type load distribution of P_0=3 MPa and a=0.5 m. The number of grids for the domain was set to 25 and 50 in the vertical and horizontal directions, respectively. For comparison with the results of the equivalent truss structure model, three-dimensional solid structural analysis was done using SAP2000^{TM} (Computers and Structures, Inc., Berkeley, California, USA) based on solid elements. The number of grids was set to 100 and 200 in the vertical and horizontal directions, respectively. Stress concentrations occurred at both the supports, as explained by Saint-Venant's principle, but the magnitudes of the stresses at the supports showed different values between the two models. However, because these types of stress concentrations are dealt with only in special issues (e.g., in structural design), only significant stresses less than 11 MPa were included (in the form of contour lines). These were made using the Kriging technique (a geo-statistical interpolation method), as shown in Figure 15a. The stress distributions of the two models over the whole domain, with the exception of the supports, were similar enough for the analysis solutions of the truss structure model to be deemed acceptable.

The other example that we analyzed was an aqueduct (a hydraulic structure), which is a structure that is frequently used in agricultural civil engineering. The shape and dimensions of the structure are shown in Figure 15b, and the following assumptions were made: a hydraulic pressure of 9.8 kN/m^3, soil pressure of 19.6 kN/m^3, elasticity modulus of 200 GPa, and thickness of 0.02 m. It was also assumed that the hydraulic and earth pressures were applied simultaneously to maintain a maximum loading state. The size of the unit grid was set to 0.005 m for both the truss structure and the solid structure analysis. Stress concentrations occurred at both the supports and the haunches, as shown in Figure 15b. In the case of a simply supported beam under a sine load, only significant stresses less than 196 GPa were plotted. For both of the examples, the equivalent truss structure model provides acceptable results that satisfy the requirements for structural analysis.

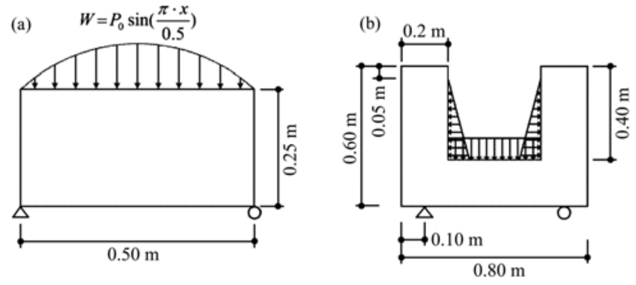
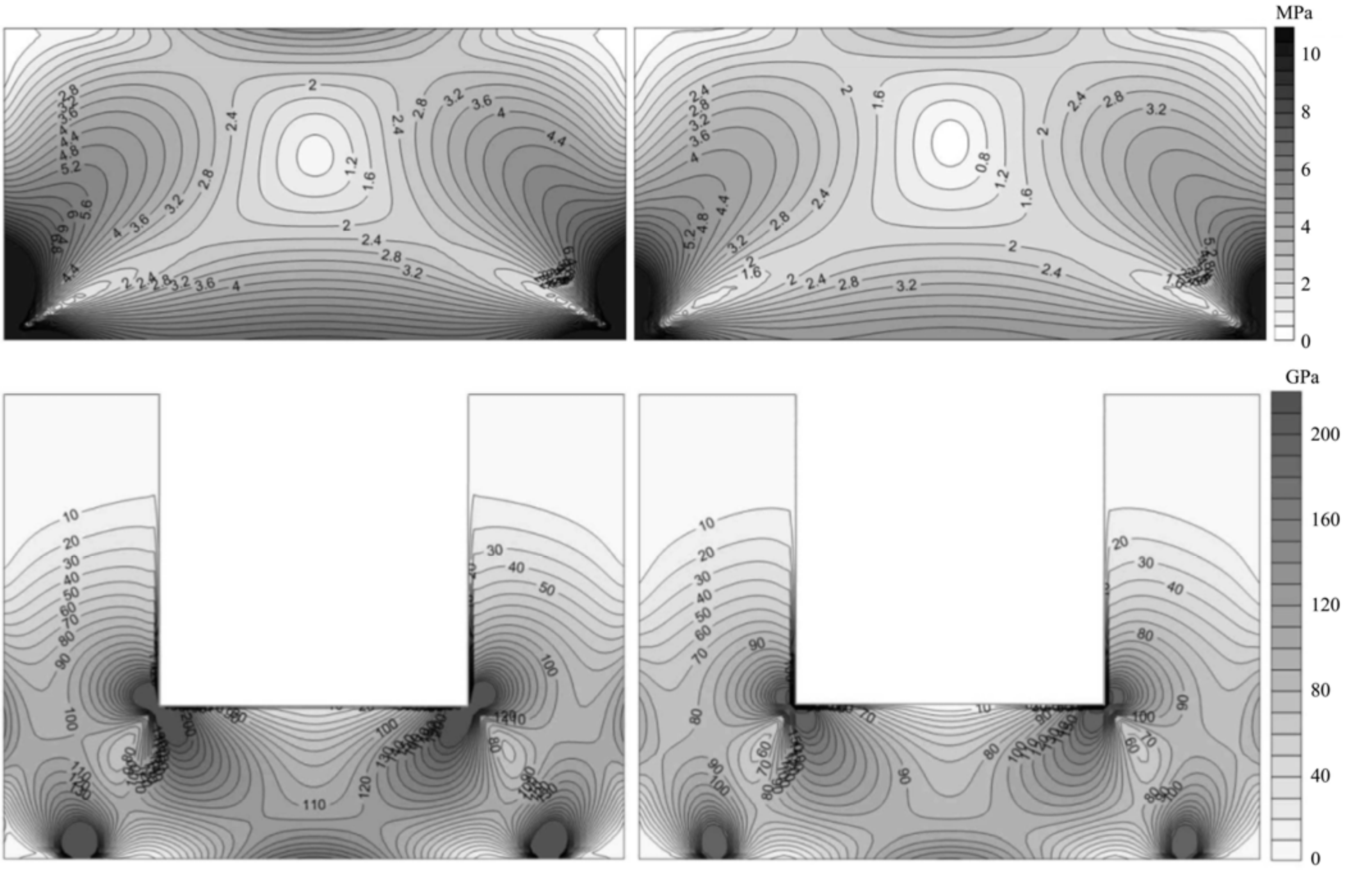


Figure 14 (a) Simply supported beam and (b) agricultural aqueduct under various loads



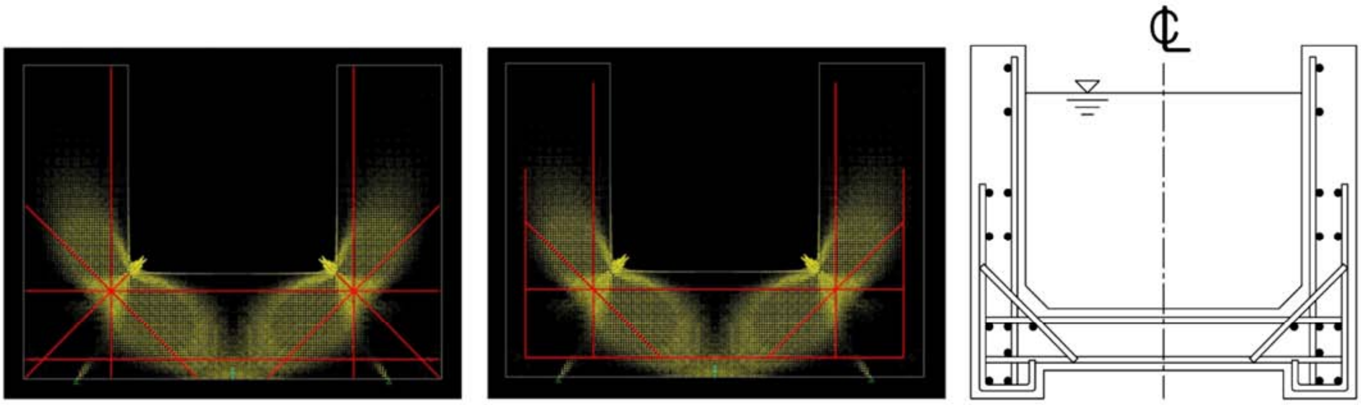
a. Solid structure model b. Truss structure model

Figure 15 von Misses stress distributions of a simply supported beam and an agricultural aqueduct

4.5 Reinforcement arrangement

A reinforced concrete (RC) structure is composed of a composite material, in which reinforcing bars resist tensile stresses and concrete resists compressive stresses. In other words, the solid structure can be divided into two regions: compressive (strut) and tensile (tie) areas, which can be replaced by the compressive and tensile elements of the equivalent truss structure model. If all of the truss elements are under compressive and near-zero forces are eliminated, the main tensile stress paths can be easily found by analyzing the remaining elements. For example, the truss elements with only tensile stresses in the agricultural aqueduct are colored yellow (Figure 16a). The principal stress paths of tensile stresses can be determined by following the truss elements with high tensile stresses; reinforced bars can be placed along these pathways. However, because these reinforcing bars should be connected to each other while also maintaining a minimum depth from the surface of the concrete, their placement should be practically decided, as shown in

Figure 16b. These results are exactly the same as the standard drawings of steel reinforcement based on the criteria of the Korea Rural Community Corporation (KRC) in South Korea. In the construction specifications of the KRC, the top and diagonal bars in the slabs of agricultural aqueducts are referred to as assembling bars. However, the truss structure model shows that the bars also play important roles as components that resist the main stresses.



a. Main paths (red color) of tensile stresses b. Practical placement of reinforced bars c. Standard drawing of the steel reinforcement

Figure 16 Truss elements (yellow color) with only tensile stresses in the agricultural aqueduct are shown

Details concerning the reinforcement of a simply supported beam are shown in Figure 17. The purpose of the longitudinal and bent-up bars located at the bottom is to resist tension that occurs in the bottom of the beam. The top bars are required for the assembly of the remaining reinforcement bars. The bent-up bars are used to resist diagonal tension caused by the combination of tension and shear stresses near the ends of the span.

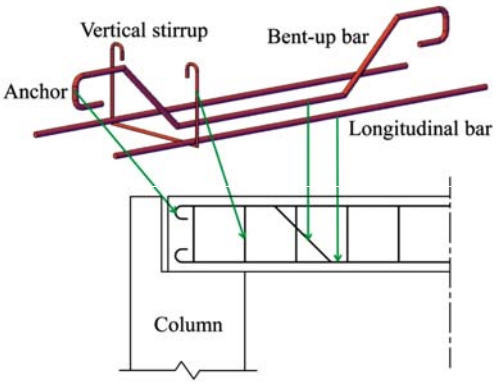
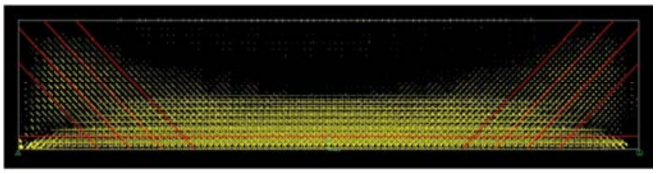
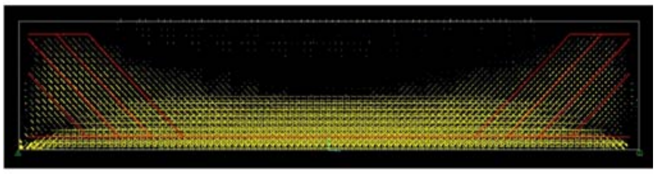


Figure 17 Typical arrangement details of the reinforcement bars in a simply supported beam

The arrangement of the reinforced bars that is required in a simply supported beam with a point load at the center of beam was also obtained by the same method mentioned above (Figure 18b). Without considering shear stresses, the equivalent truss structure model can intuitionally suggest the appropriate arrangements of the reinforcement bars.



a. Main paths (red color) of tensile stresses



b. Practical placement of the reinforced bars

Figure 18 Truss elements (yellow color) with only tensile stresses in a simply supported beam are shown

5 Conclusions

In order to have identical behavior with a continuum structure for a regular truss structure model, a two-dimensional linear elastic state was developed based on the law of the conservation of energy. The advanced model agreed well with the structural analysis that was based on the three-dimensional finite element method. Additionally, our proposed model was practically useful for problems concerning specific areas (e.g., the arrangements of reinforcement bars), which are related to the main pathways of stresses as opposed to all of the areas, due to its accuracy. In the future, we expect that the equivalent truss structure model, which possesses definite advantages in simulating crack propagation by using simple criteria to check for tensile or compressive failure, will dramatically decrease the computing times required for the reliability analysis of complex structures in areas related to agricultural engineering.

Acknowledgment

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